

Thermodynamics in $f(T)$ Gravity and Corrected Entropies

M. Sharif *and Shamaila Rani[†]

Department of Mathematics, University of the Punjab,
Quaid-e-Azam Campus, Lahore-54590, Pakistan.

Abstract

This paper is devoted to study the generalized second law of thermodynamics in $f(T)$ gravity. We use quantum corrections such as power-law and logarithmic corrected entropies to the horizon entropy along with Gibbs' equation in the thermal equilibrium state. We derive $f(T)$ model by taking into account a power-law scale factor through the first modified Friedmann equation which obeys the condition for a realistic model. Two types of horizons, i.e., Hubble and event horizons are used to check the validity of the generalized second law of thermodynamics with corrected entropies. We conclude that this law holds with a specific range of entropy parameter on both horizons in the case of power-law corrected entropy, while it violates for all values of entropy parameter on both horizons for logarithmic corrected entropy.

Keywords: $f(T)$ gravity; Generalized second law of thermodynamics.

PACS: 04.50.kd; 05.70.-a.

1 Introduction

The modified theories of gravity are well-known for a combined motivation coming from cosmology, high-energy physics, astrophysics and quantum

*msharif.math@pu.edu.pk

[†]shamailatoor.math@yahoo.com

physics. In these theories, the $f(T)$ gravity [1]-[3] has attained a lot of interest for describing the present accelerated state of the universe along with some attractive features geometrically and physically. It is a straightforward modification of teleparallel theory [4]-[6] by introducing an arbitrary function $f(T)$ in its Lagrangian density instead of torsion scalar T . This theory deals with torsion via Weitzenböck connection (having zero curvature) instead of Levi-Civita connection responsible for curvature. It is easy to tackle in the sense that the torsion scalar involves products of first derivatives of tetrad (dynamical field) and the field equations contain the second order.

This modified gravity has been studied extensively under many phenomena, such as, accelerated expansion of the universe [7]-[10], large-scale structure [11], cosmological perturbations [12, 13], discussion of Birkhoff's theorem [14], static spherically symmetric solutions [15], solar system constraints [16], reconstructions via scalar fields [17, 18], viability of models through cosmographic technique [19, 20], thermodynamics [21]-[23] and many more. The connection between thermodynamics and gravitation is served by the black hole thermodynamics [25, 26]. Jacobson [27] used the relation among the entropy and horizon area in thermodynamics and derived the Einstein equations. This work is extended for the curvature correction [28] to the entropy in the form of polynomial Ricci scalar in non-equilibrium thermodynamics. Bamba et al. [29] investigated that the equations of motion of modified gravity theories, particularly $f(R)$, $f(G)$, scalar Gauss-Bonnet and non-local theories are equivalent to the Clausius relation in thermodynamics. It is worthwhile to check the viability of the generalized second law of thermodynamics (GSLT) in the accelerated universe [30]-[33].

Bamba and Geng [21] explored the thermodynamics in equilibrium and non-equilibrium descriptions for apparent horizon in $f(T)$ gravity. Karami and Abdolmaleki [22] explored the validity of GSLT on Hubble horizon using power-law and exponential models. They concluded that this law holds for both models from early to present universe, while it is violated in the future epoch. Bamba et al. [34] discussed the finite time singularities, Little Rip, Pseudo-Rip cosmologies and thermodynamics for the apparent horizon bounded universe. Some people have discussed the validity of this law by introducing correction terms in entropy and horizon area in general relativity as well as in modified gravities.

Debnath et al. [35] investigated the validity of GSLT by taking power-law corrected entropy (PLCE) in equilibrium and non-equilibrium cases for apparent and event horizons in general relativity. They found some constraints

on the power-law parameter and small perturbation in de Sitter spacetime for the validity of this law. In the case of logarithmic corrected entropy (LCE), Sadjadi and Jamil [36] found that the validity occurs for positive LCE parameter and concluded that this law holds throughout the universe for spatial curvature with any dark energy model. Sharif and Jawad [37] discussed the validity of GSLT with corrected entropies for three different systems in the closed universe. Recently, Bamba et al. [23] studied the constraints on PLCE and LCE parameters to satisfy or violate the GSLT in $f(T)$ gravity by taking a system of n -component fluids in thermal equilibrium for apparent and event horizons.

In this paper, we discuss the validity of this law with PLEC and LEC for Hubble as well as event horizons in $f(T)$ gravity by constructing a realistic $f(T)$ model. The scheme of the paper is as follows. In section **2**, we provide a brief review of $f(T)$ formalism and entropy corrections. Section **3** is devoted to discuss the validity of GSLT with PLCE and LCE for Hubble as well as event horizons in equilibrium state. The last section summarizes the results.

2 Brief Review

Here we provide briefly the formalism of $f(T)$ gravity, its field equations and some entropy corrections to the horizon entropy.

2.1 $f(T)$ Gravity and Field Equations

The two connected parts/structures of a manifold involve Riemannian structure with a definite metric and non-Riemannian structure having torsion or non-metricity. The Weitzenböck spacetime is defined by the second structure which has zero Riemannian tensor but non-zero torsion based on the tetrad field. This was originally proposed by Einstein to unify the electromagnetism with gravity and introduced teleparallel theory of gravity. The dynamical tetrad field $h_a(x^\mu)$ is an orthonormal basis for the tangent space at each point of the manifold [24]. This field is analyzed by tetrad components h_a^μ ($\mu, a = 0, 1, 2, 3$) in the coordinate basis $h_a = h_a^\mu \partial_\mu$, related by $h_\mu^a h_b^\mu = \delta_b^a$ and $h_\mu^a h_a^\nu = \delta_\mu^\nu$. We denote the coordinates on the manifold by Greek indices while the Latin alphabets refer to the tangent space. The metric tensor is obtained by the dual tetrad components as $g_{\mu\nu} = \eta_{ab} h_\mu^a h_\nu^b$.

The Weitzenböck connection is defined from tetrad as $\tilde{\Gamma}^\rho_{\mu\nu} = h^\rho_a \partial_\nu h^a_\mu$, yielding the following antisymmetric torsion tensor

$$T^\rho_{\mu\nu} = \tilde{\Gamma}^\rho_{\nu\mu} - \tilde{\Gamma}^\rho_{\mu\nu} = h^\rho_a (\partial_\mu h^a_\nu - \partial_\nu h^a_\mu).$$

The antisymmetric superpotential $S_\rho^{\mu\nu}$ and contorsion $K^{\mu\nu}_\rho$ tensors are

$$S_\rho^{\mu\nu} = \frac{1}{2}(K^{\mu\nu}_\rho + \delta^\mu_\rho T^{\theta\nu}_\theta - \delta^\nu_\rho T^{\theta\mu}_\theta), \quad K^{\mu\nu}_\rho = -\frac{1}{2}(T^{\mu\nu}_\rho - T^{\nu\mu}_\rho - T_\rho^{\mu\nu}),$$

which are used to define the torsion scalar as

$$T = T^\rho_{\mu\nu} S_\rho^{\mu\nu}. \quad (1)$$

The action of $f(T)$ gravity is given by

$$S = \frac{1}{2\kappa^2} \int d^4x [hf(T) + \mathcal{L}_m], \quad (2)$$

where $h = \sqrt{-g}$, $\kappa^2 = 8\pi G$, G is the gravitational constant and \mathcal{L}_m is the matter Lagrangian density inside the universe. The corresponding field equations are obtained by varying this action with respect to tetrad as [38]

$$[h^{-1}\partial_\mu(hS_a^{\mu\nu}) + h^\lambda_a T^\rho_{\mu\lambda} S_\rho^{\nu\mu}]f_T + S_a^{\mu\nu}\partial_\mu(T)f_{TT} + \frac{1}{4}h^\nu_a f = \frac{1}{2}\kappa^2 h^\rho_a T^\nu_\rho, \quad (3)$$

where $f_T = df/dT$, $f_{TT} = d^2f/dT^2$ and T^ν_ρ is the energy-momentum tensor of perfect fluid.

For the flat FRW universe, we take tetrad components as $h^a_\nu = \text{diag}(1, a, a, a)$ [17, 18]. The corresponding modified Friedmann equations are

$$12H^2 f_T + f = 2\kappa^2 \rho, \quad (4)$$

$$48\dot{H}H^2 f_{TT} - (12H^2 + 4\dot{H})f_T - f = 2\kappa^2 p, \quad (5)$$

where ρ and p are the total energy density and pressure of the universe and $H = \dot{a}/a$. The above field equations can be written as

$$\frac{3H^2}{\kappa^2} = \rho, \quad -\frac{2\dot{H}}{\kappa^2} = \rho + p, \quad (6)$$

where $\rho = \rho_m + \rho_T$ and $p = p_m + p_T$. We assume here the pressureless (dust) matter, i.e., $p_m = 0$ and ρ_T , p_T are torsion contributions given by

$$\rho_T = \frac{1}{2\kappa^2}(-12H^2 f_T - f + 6H^2), \quad (7)$$

$$p_T = -\frac{1}{2\kappa^2}(48\dot{H}H^2 f_{TT} - (12H^2 + 4\dot{H})f_T - f + 6H^2 + 4\dot{H}). \quad (8)$$

The corresponding energy conservations are

$$\dot{\rho}_m + 3H\rho_m = 0, \quad \dot{\rho}_T + 3H(\rho_T + p_T) = 0. \quad (9)$$

For dust matter, it yields

$$\rho_m = \rho_{m0}a^{-3}, \quad (10)$$

where ρ_{m0} is an arbitrary constant.

We assume here the power-law scale factor as [39]-[41]

$$a(t) = a_0(t_s - t)^{-b}, \quad b > 0, \quad t_s \geq t, \quad (11)$$

where a_0 is the present-day value of the scale factor. This scale factor indicates the superaccelerated universe with a Big Rip singularity at $t = t_s$. Using this scale factor, the Hubble parameter, torsion scalar and \dot{H} become

$$H = \frac{b}{t_s - t}, \quad T = -\frac{6b^2}{(t_s - t)^2}, \quad \dot{H} = \frac{b}{(t_s - t)^2}. \quad (12)$$

Inserting these values in Eq.(4), we obtain

$$f(T) = c \left(-\frac{T}{6b^2} \right)^{\frac{1}{2}} + \frac{2\kappa^2\rho_{m0}}{a_0^3(3b+1)} \left(-\frac{6b^2}{T} \right)^{\frac{3b}{2}}, \quad (13)$$

where c is an integration constant which can be determined by imposing a suitable boundary condition. This model satisfies the condition of a realistic model, i.e., $\frac{f}{T} \rightarrow 0$ [19, 20] as $T \rightarrow \infty$ at high redshift, representing an accelerated expansion of the universe which is consistent with the primordial nucleosynthesis and cosmic microwave background constraints. To determine c , we impose the condition on gravitational constant G . For non-linear $f(T)$, Eq.(4) implies an effective gravitational constant (time dependent), G_{eff} instead of G ($\kappa^2 = 8\pi G$). It must reduce to the present day value of G for linear $f(T)$ which yields the condition $f_T(T_0) = 1$, where $T_0 = -6H_0^2$ and H_0 is the present day value of Hubble parameter. Using the model (13) in this condition, it follows that

$$c = 12bH_0 \left[\frac{b\kappa^2\rho_{m0}}{2a_0^3(3b+1)H_0^2} \left(\frac{b}{H_0} \right)^{3b} - 1 \right]. \quad (14)$$

2.2 Corrected Entropies

The correction terms in the entropy-area relationship are widely discussed to study the thermodynamical systems. The entropy of the horizon is proportional to the area of the horizon ($S \propto A$) in the Einstein gravity. If we modify the action of gravity theory by adding some extra curvature terms, it changes the entropy-area relation, e.g., it takes the form $S \propto A f_R$ [42] in $f(R)$ theory, where f_R is the derivative of arbitrary function f with respect to the Ricci scalar R . This relationship is affected by some field anomalies and gravitational fluctuations based on black hole physics. To deal with these fluctuations, quantum corrections to the semi-classical entropy law have been introduced in the form of power-law and logarithmic.

The entanglement of quantum fields in and out the horizon generate the corrections to the entropy such as a power-corrected area term in the entropy expression. The power-law corrected entropy takes the form [23, 43]

$$S_X = \frac{A}{4G} \left(1 - K_\alpha A^{1-\frac{\alpha}{2}}\right), \quad K_\alpha = \frac{\alpha(4\pi)^{\frac{\alpha}{2}-1}}{(4-\alpha)r_c^{2-\alpha}} \quad (15)$$

where $A = 4\pi R_X^2$, R_X is the radius of an arbitrary horizon X , α is a dimensionless constant which should be greater than zero for entropy to be well-defined and $r_c \sim H_0^{-1}$ is the crossover scale [44]. The correction term in Eq.(15) is the result of wave function of the field which is the entanglement of ground and excited states. The excited state contributes to the correction while the ground state entanglement entropy satisfies the black hole entropy-area relationship. For higher excitation states, the correction term is more significant and it falls off rapidly with increments in area, i.e., in the semi-classical limit (large area), the entropy-area law is recovered. The curvature correction in the Einstein-Hilbert action is formed due to quantum corrections into the entropy-area relationship and vice versa. This leads to the logarithmic corrected entropy [23, 45, 46]

$$S_X = \frac{A}{4G} + \beta \log \left(\frac{A}{4G} \right) + \gamma, \quad (16)$$

where β and γ are dimensionless constants whose exact values are not yet known. These corrections arise due to mass-charge, quantum and thermal equilibrium fluctuations in loop quantum gravity.

3 Thermodynamics

The generalized second law of thermodynamics states that the sum of entropy of the horizon and entropy of total matter inside the horizon does not decrease with time. The Clausius relation using the first law of thermodynamics is found to be $-dE = T_X dS_X$, where $S_X = \frac{A}{4G}$ is the Bekenstein entropy (entropy-area relation) and $T_X = \frac{1}{2\pi R_X}$ is the Hawking temperature. Miao et al. [47] found that the first law of thermodynamics violates in $f(T)$ gravity due to lack of local Lorentz invariance which leads to some degrees of freedom and results an additional entropy production term S_P . In order to reduce the degrees of freedom, they get a condition $\epsilon = \frac{4f_{TT}(0)}{f(0)} < 0$, where ϵ denotes the violation of local Lorentz invariance. This parameter ϵ and f_{TT} should be very small to be consistent with the experiments. Thus, the first law of thermodynamics holds if f_{TT} is very small and entropy horizon becomes $S_X = \frac{Af_T}{4G}$ with zero S_P . The entropy-area relation is modified to $A \rightarrow Af_T$, which leads to the modification of power-law (PX) and logarithmic (LX) corrected entropies (15) and (16), given by [23]

$$S_{PX} = \frac{Af_T}{4G} (1 - K_\alpha (Af_T)^{1-\frac{\alpha}{2}}), \quad S_{LX} = \frac{Af_T}{4G} + \beta \log \left(\frac{Af_T}{4G} \right) + \gamma. \quad (17)$$

The Gibbs' equation is used to find the rate of change of normal entropy S_I of the horizon

$$\dot{S}_I + \dot{S}_P = \frac{1}{T_X} \left(\frac{dE_I}{dt} + p \frac{dV}{dt} \right), \quad (18)$$

where $E_I = \rho V$, $V = \frac{4}{3}\pi R_X^3$ is the volume of the horizon. Inserting these values in this equation, it follows that

$$\dot{S}_I + \dot{S}_P = 8\pi^2 R_X^3 (\dot{R}_X - H R_X) (\rho + p). \quad (19)$$

The time derivative of power-law and logarithmic corrected entropies become

$$\dot{S}_{PX} = \frac{2\pi R_X}{G} (\dot{R}_X f_T - 6H \dot{H} R_X f_{TT}) [1 - (2 - \frac{\alpha}{2}) K_\alpha (4\pi R_X^2 f_T)^{1-\frac{\alpha}{2}}] \quad (20)$$

$$\dot{S}_{LX} = 2 \left(\frac{\pi R_X}{G} + \frac{\beta}{R_X f_T} \right) (\dot{R}_X f_T - 6H \dot{H} R_X f_{TT}). \quad (21)$$

For the validity of GSLT, we first see the behavior of the second derivative of the model (13) with $a(t) = \frac{a_0}{1+z}$, it follows that

$$f_{TT} = \frac{\kappa^2 \rho_{m0} (3b+2)}{24a_0^3 b^3 (3b+1)} (1+z)^{3+\frac{4}{b}} - \frac{c}{144b^4} (1+z)^{\frac{3}{b}}. \quad (22)$$

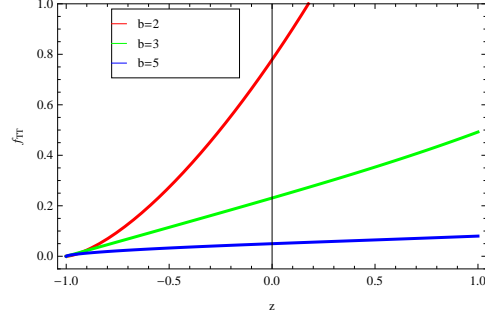


Figure 1: Plot of f_{TT} versus z .

Its plot versus z is shown in Figure 1 for $b = 2, 3, 5$ and using $\kappa^2 = \rho_{m0} = a_0 = 1$, $H_0 = 74.2 \text{ km s}^{-1} \text{ Mpc}^{-1}$. The graph indicates that $f_{TT} \ll 1$ in the range $z < 0.18$ for $b = 2$ whereas it satisfies for $b = 3, 5$ and all values of z . Thus we take the entropy production term to be zero in Eq.(19).

In the following, we check the validity of GSLT for Hubble and event horizons for both corrected entropies.

3.1 Hubble Horizon

Consider the boundary of thermal system of the FRW spacetime covered by the Hubble horizon in equilibrium state. It is the reduction of apparent horizon for flat space [48]. The radius of Hubble horizon and its time derivative are given by

$$R_H = \frac{1}{H}, \quad \dot{R}_H = -\frac{\dot{H}}{H^2}. \quad (23)$$

Power-law Corrected Entropy

Replacing X by H in Eqs.(19) and (20), the time derivative of total entropy for Hubble horizon, i.e., $\dot{S}_{PLCE} = \dot{S}_{PH} + \dot{S}_I$ becomes

$$\begin{aligned} \dot{S}_{PLCE} = & -\frac{8\pi^2}{H^3} \left(1 + \frac{\dot{H}}{H^2} \right) \left[\rho_{m0} a^{-3} + \frac{1}{4\pi G} (2\dot{H}T f_{TT} + \dot{H}(f_T - 1)) \right] \\ & - \frac{2\pi}{GH} \left(\frac{\dot{H}}{H^2} f_T + 6\dot{H} f_{TT} \right) \left[1 - \left(2 - \frac{\alpha}{2} \right) K_\alpha \left(\frac{4\pi}{H^2} f_T \right)^{1-\frac{\alpha}{2}} \right]. \end{aligned} \quad (24)$$

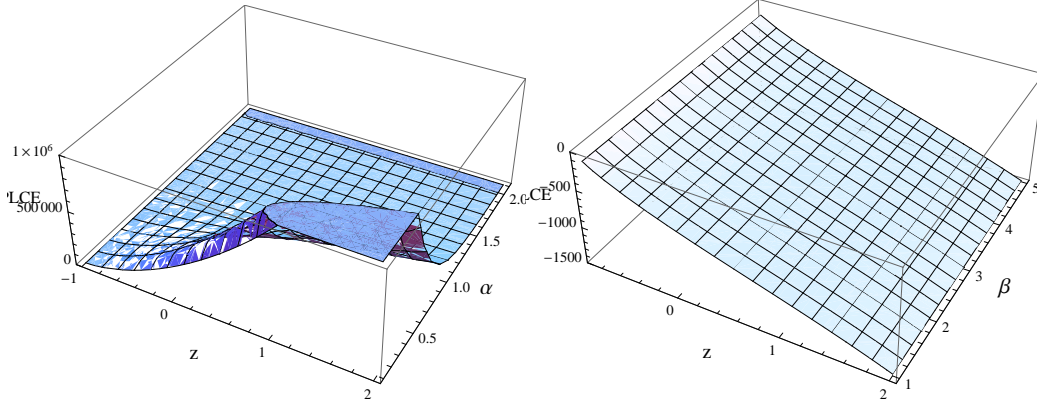


Figure 2: Plots of the rate of change of total entropy versus redshift and model parameters for Hubble horizon. The left graph is for power-law corrected entropy and the right is for logarithmic corrected entropy.

Using Eqs.(11)-(13), we obtain

$$\begin{aligned} \dot{S}_{PLCE} = & -\frac{\pi(1+z)^{\frac{1}{b}}}{Gb^3} \left[2(1+b) - \frac{\kappa^2 \rho_{m0}(4+3b)(1+z)^{3+\frac{2}{b}}}{2a_0^3(1+3b)} + \frac{c(1+z)^{\frac{1}{b}}}{4b} \right. \\ & \times \left. \left(1 - \frac{\alpha(4\alpha)^{\frac{\alpha}{2}-1}}{2H_0^{\alpha-2}} \left(\frac{2\pi\kappa^2 \rho_{m0}(1+z)^{3+\frac{4}{b}}}{a_0^3 b^3(1+3b)} - \frac{c\pi(1+z)^{\frac{3}{b}}}{3b^4} \right)^{1-\frac{\alpha}{2}} \right) \right], \end{aligned} \quad (25)$$

which is the time derivative of the total entropy with power-law correction for Hubble horizon with $f(T)$ model (13) in terms of z . Its plot versus z and α is shown in Figure 2 (left) keeping the same values of constants with $b = 3$. Initially, the graph represents large positive values of \dot{S}_{PLCE} for higher values of z , then it decays and remains positive for the present universe ($z = 0$) towards future ($z < 0$). For $\alpha \leq 2$, the graph remains positive, otherwise shows negative behavior. Thus, the GSLT is valid for all values of z with $\alpha \leq 2$, while it violates for $\alpha > 2$.

Logarithmic Corrected Entropy

The time derivative of total entropy for Hubble horizon ($X \rightarrow H$) with logarithmic correction using Eqs.(19) and (21), i.e., $\dot{S}_{LCE} = \dot{S}_{LH} + \dot{S}_I$ takes

the form

$$\begin{aligned}\dot{S}_{LCE} = & -2 \left(\frac{\pi}{GH} + \frac{\beta H}{f_T} \right) \left(\frac{\dot{H}}{H^2} f_T - 6 \dot{H} f_{TT} \right) - \frac{8\pi^2}{H^3} \left(1 + \frac{\dot{H}}{H^2} \right) \\ & \times \left[\rho_{m0} a^{-3} + \frac{1}{4\pi G} (2\dot{H} T f_{TT} + \dot{H} (f_T - 1)) \right].\end{aligned}\quad (26)$$

Inserting $f(T)$ model along with scale factor and Hubble horizon in the above equation, we obtain

$$\begin{aligned}\dot{S}_{LCE} = & -2 \left[\frac{\pi(1+z)^{\frac{1}{b}}}{Gb} + \frac{\beta b^2}{(1+z)^{\frac{2}{b}}} \left(\frac{\kappa^2 \rho_{m0}}{2a_0^3(1+3b)} (1+z)^{3+\frac{1}{b}} - \frac{c}{12b} \right)^{-1} \right] \\ & \times \left[\frac{\kappa^2 \rho_{m0}(4+3b)}{4a_0^3 b^2(1+3b)} (1+z)^{3+\frac{2}{b}} - \frac{c(1+z)^{\frac{1}{b}}}{8b^3} \right] + \frac{2\pi(1+b)(1+z)^{\frac{1}{b}}}{Gb^3}\end{aligned}\quad (27)$$

Figure 2 (right) represents its behavior versus z and β . It indicates negative behavior for all values of z and β . As z decreases, \dot{S}_{LEC} increases and gets closer to positive values for present and future epochs but remains negative. Thus, GSLT does not hold for logarithmic entropy correction.

3.2 Event Horizon

Now we assume the event horizon [49] as boundary of thermal equilibrium system whose existence is related to the convergence of the following integral

$$R_E = a \int_t^\infty \frac{dt}{a}, \quad \dot{R}_E = H R_E - 1. \quad (28)$$

It is the distance of light traveling from present time to infinity and we replace ∞ by t_s for Big Rip future time singularity.

Power-law Corrected Entropy

Using Eqs.(19) and (20), replacing the arbitrary horizon X by event horizon and adding the resulting equations, it yields

$$\dot{S}_{PLCE} = -8\pi^2 \left(a \int_t^\infty \frac{dt}{a} \right)^3 \left(\rho_{m0} a^{-3} + \frac{1}{4\pi G} (2\dot{H} T f_{TT} + \dot{H} f_T - \dot{H}) \right)$$

$$\begin{aligned}
& + \frac{2\pi}{G} a \int_t^\infty \frac{dt}{a} \left[\left(H a \int_t^\infty \frac{dt}{a} - 1 \right) f_T - 6a \int_t^\infty \frac{dt}{a} H \dot{H} f_{TT} \right] \\
& \times \left[1 - \left(2 - \frac{\alpha}{2} \right) K_\alpha \left(4\pi \left(a \int_t^\infty \frac{dt}{a} \right)^2 f_T \right)^{1-\frac{\alpha}{2}} \right]. \tag{29}
\end{aligned}$$

This is the time derivative of total entropy with power-law corrected entropy for event horizon. Using $f(T)$ model, $a(t)$ and H in terms of z , we obtain

$$\begin{aligned}
\dot{S}_{PLCE} &= \frac{2\pi b}{G(1+b)^3} (1+z)^{\frac{1}{b}} - \left[\frac{\pi \kappa^2 \rho_{m0} (4+3b)}{2G a_0^3 b (1+b)^2 (1+3b)} (1+z)^{3+\frac{3}{b}} \right. \\
& - \left. \frac{2\pi c (1+z)^{\frac{2}{b}}}{8G b^2 (1+b)^2} \right] \left[1 - \frac{\alpha (4\alpha)^{\frac{\alpha}{2}-1}}{2H_0^{\alpha-2}} \left(\frac{2\pi \kappa^2 \rho_{m0} (1+z)^{3+\frac{4}{b}}}{a_0^3 b (1+b)^2 (1+3b)} \right. \right. \\
& - \left. \left. \frac{c\pi (1+z)^{\frac{3}{b}}}{3b^2 (1+b)^2} \right)^{1-\frac{\alpha}{2}} \right]. \tag{30}
\end{aligned}$$

Its plot versus z and α is shown in the left panel of Figure 3 which expresses the same behavior as for the Hubble horizon. The only difference is the values of time derivative of total entropies in the corresponding intervals of z . The GSLT satisfies for all values of z with $\alpha \leq 2$ for power-law corrected entropy.

Logarithmic Corrected Entropy

For event horizon ($X \rightarrow E$ in Eqs.(19) and (21)), the rate of change of total entropy becomes

$$\begin{aligned}
\dot{S}_{PLCE} &= -8\pi^2 \left(a \int_t^\infty \frac{dt}{a} \right)^3 \left(\rho_{m0} a^{-3} + \frac{1}{4\pi G} (2\dot{H} T f_{TT} + \dot{H} f_T - \dot{H}) \right) \\
& + 2 \left(\frac{\pi a}{G} \int_t^\infty \frac{dt}{a} + \frac{\beta}{a \int_t^\infty \frac{dt}{a} f_T} \right) \left[\left(H a \int_t^\infty \frac{dt}{a} - 1 \right) f_T - 6a H \right. \\
& \times \left. \int_t^\infty \frac{dt}{a} \dot{H} f_{TT} \right], \tag{31}
\end{aligned}$$

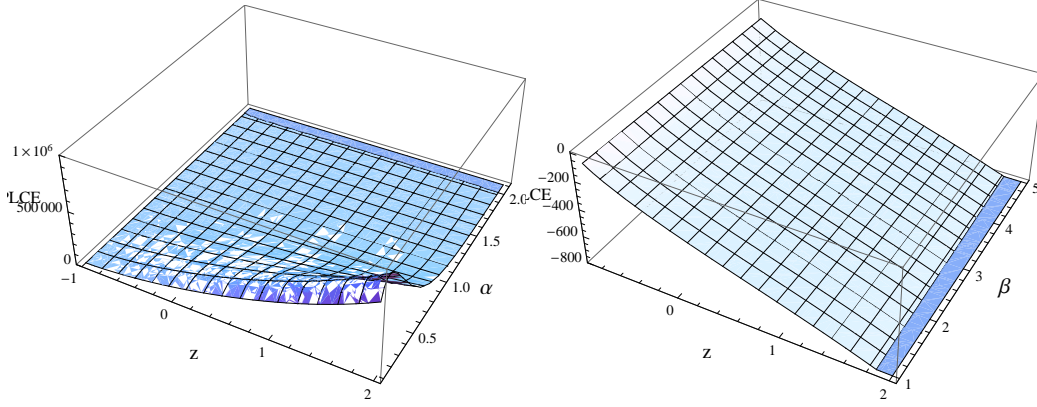


Figure 3: Plots of the rate of change of total entropy versus redshift and model parameters for event horizon. The left graph is for power-law corrected entropy and the right is for logarithmic corrected entropy.

yielding

$$\begin{aligned} \dot{S}_{LCE} = & -2 \left[\frac{\pi(1+z)^{\frac{1}{b}}}{G(1+b)} + \frac{2b(1+b)\beta}{(1+z)^{\frac{2}{b}}} \left(\frac{\kappa^2 \rho_{m0}}{a_0^3(1+3b)} (1+z)^{3+\frac{1}{b}} - \frac{c}{6b} \right)^{-1} \right] \\ & \times \left[\frac{\kappa^2 \rho_{m0}(4+3b)(1+z)^{3+\frac{2}{b}}}{4a_0^3 b(1+b)(1+3b)} - \frac{c(1+z)^{\frac{1}{b}}}{8b^2(1+b)} \right] + \frac{2\pi b(1+z)^{\frac{1}{b}}}{G(1+b)^3}. \end{aligned} \quad (32)$$

Figure 3 (right) shows its graph versus z and β . This also represents the same behavior of total entropy for the logarithmic correction with the same range of z and β as for the Hubble horizon. Thus GSLT also violates for the total entropy having logarithmic correction for the event horizon.

4 Concluding Remarks

In this paper, we have discussed the validity of GSLT in the context of $f(T)$ gravity in FRW universe. We have taken the corrected entropies such as, PLCE and LCE to the entropy-area relationship. A power-law scale factor is chosen to construct the $f(T)$ model and integration constant is found through a boundary condition on G_{eff} . This model satisfies the condition for a realistic model. We have checked the behavior of the second derivative of $f(T)$ model in order to meet the first law of thermodynamics. The validity

of GSLT with corrected entropies are investigated through graphical representation for two horizons, Hubble and event horizons in equilibrium state. The results for both these horizons are summarized as follows.

The $f(T)$ model satisfies the condition, $f_{TT} \ll 1$ which leads to take entropy production term to be zero. The time derivative of total entropy with PLCE for Hubble and event horizons represents positive behavior versus z for $h = 3$ within a specific range of PLCE parameter $\alpha \leq 2$. The only difference comes in the values of PLCE in the corresponding intervals of z . Thus GSLT for this corrected entropy satisfies in the underlying scenario. The LCE for both horizons shows the same behavior as the violation of GSLT throughout the spacetime for z and LCE parameter β with $h = 3$. We have assumed $\beta > 0$ [50] which leads to positive contribution to the entropy of the system. The values of these correction parameters are not sensitive corresponding to the obtained behavior of the rate of change of total entropy. However, for very high value of h , the rate of change of total entropy gives positive results for LCE while it becomes negative for PLCE.

Bamba et al. [23] studied the validity of GSLT in $f(T)$ gravity (with $F(T) = T + f(T)$) generally in thermal equilibrium for apparent and event horizons. They assumed the basic requirement $f_{TT} \ll 1$ to hold the first law of thermodynamics in addition to $f_T > 0$ to constrain the PLCE and LCE parameters. They concluded that for PLCE and LCE, GSLT satisfies for any value of correction parameters for Hubble horizon. In case of event horizon, the validity of GSLT depends upon the time derivative of event horizon for both entropy corrections. These results hold regardless of any choice of $f(T)$ model. However, we have obtained constraints on correction parameters for a constructed $f(T)$ model to check the validity of GSLT. Also, in a recent paper [51], we have discussed the validity of this law incorporating the nonlinear electrodynamics and dust matter in $f(T)$ gravity with two types of scale factor for Hubble and event horizons. It was shown that this law holds in this case only in the early universe and violates for the present and future epochs for both horizons with power-law scale factor. Here the PLCE provides the validity of GSLT for the same scale factor and violation turns out for LCE. It is interesting to mention here that all our results become equivalent to the results of [51] for zero entropy correction terms as well as magnetic field.

References

- [1] R. Ferraro, F. Fiorini, Phys. Rev. D **75**, 084031 (2007).
- [2] G.R. Bengochea, R. Ferraro, Phys. Rev. D **79**, 124019 (2009).
- [3] E.V. Linder, Phys. Rev. D **81**, 127301 (2010).
- [4] J.W. Maluf, J. Math. Phys. **35**, 335 (1994).
- [5] V.C. de Andrade, J.G. Pereira, Phys. Rev. D **56**, 4689 (1997).
- [6] G.G.L. Nashed, Gen. Relativ. Gravit. **34**, 1047 (2002).
- [7] M. Sharif, S. Rani, Phys. Scr. **84**, 055005 (2011).
- [8] M. Sharif, S. Rani, Mod. Phys. Lett. A **26**, 1657 (2011).
- [9] R.J. Yang, Eur. Phys. J. C **71**, 1797 (2011).
- [10] R. Myrzakulov, Eur. Phys. J. C **71**, 1752 (2011).
- [11] B. Li, T.P. Sotiriou, J.D. Barrow, Phys. Rev. D **83**, 104017 (2011).
- [12] J.B. Dent, S. Dutta, E.N. Saridakis, J. Cosmol. Astropart. Phys. **01**, 009 (2011).
- [13] S-H. Chen, J.B. Dent, S. Dutta, E.N. Saridakis, Phys. Rev. D **83**, 023508 (2011).
- [14] X-H. Meng, Y-B. Wang, Eur. Phys. J. C **71**, 1755 (2011).
- [15] T. Wang, Phys. Rev. D **84**, 024042 (2011).
- [16] L. Iorio, E.N. Saridakis, Mon. Not. Roy. Astron. Soc. **427**, 1555 (2012).
- [17] M. Sharif, S. Rani, Astrophys. Space Sci. **345**, 217 (2013).
- [18] M. Jamil, D. Momeni, R. Myrzakulov, Eur. Phys. J. C **72**, 2057 (2012).
- [19] S. Capozziello, V.F. Cardone, H. Farajollahi, A. Ravanpak, Phys. Rev. D **84**, 043527 (2011).

- [20] K. Bamba, S. Capozziello, S. Nojiri, S.D. Odintsov, *Astrophys. Space Sci.* **342**, 155 (2012).
- [21] K. Bamba, C.Q. Geng, *J. Cosmol. Astropart. Phys.* **11**, 008 (2011).
- [22] K. Karami, A. Abdolmaleki, *J. Cosmol. Astropart. Phys.* **04**, 007 (2012).
- [23] K. Bamba, M. Jamil, D. Momeni, R. Myrzakulov, *Astrophys. Space Sci.* **344**, 259 (2013).
- [24] K. Hayashi, T. Shirafuji, *Phys. Rev. D* **19**, 3524 (1979).
- [25] J.M. Bardeen, B. Carter, S. Hawking, *Commun. Math. Phys.* **31**, 161 (1973).
- [26] G. Gibbons, S. Hawking, *Phys. Rev. D* **15**, 2738 (1977).
- [27] T. Jacobson, *Phys. Rev. Lett.* **75**, 1260 (1995).
- [28] C. Eling, R. Guedens, T. Jacobson, *Phys. Rev. Lett.* **96**, 121301 (2006).
- [29] K. Bamba, C-Q. Geng, S. Nojiri, S.D. Odintsov, *Europhys. Lett.* **89**, 50003 (2010).
- [30] H.M. Sadjadi, *Phys. Rev. D* **76**, 104024 (2007).
- [31] M. Jamil, E.N. Saridakis, M.R. Setare, *J. Cosmol. Astropart. Phys.* **11**, 032 (2010).
- [32] A. Sheykhi, *Eur. Phys. J. C* **69**, 265 (2010).
- [33] K. Karami, S. Ghaffari, *Phys. Lett. B* **688**, 125 (2010).
- [34] K. Bamba, R. Myrzakulov, S. Nojiri, S.D. Odintsov, *Phys. Rev. D* **85**, 104036 (2012).
- [35] U. Debnath, *et al.*, *Eur. Phys. J. C* **72**, 1875 (2012).
- [36] H.M. Sadjadi, M. Jamil, *Europhys. Lett.* **92**, 69001 (2010).
- [37] M. Sharif, A. Jawad, *Int. J. Mod. Phys. D* **22**, 1350014 (2013).
- [38] R. Ferraro, F. Fiorini, *Phys. Lett. B* **702**, 75 (2011).

- [39] S. Nojiri, S.D. Odintsov, S. Tsujikawa, Phys. Rev. D **71**, 063004 (2005).
- [40] H.M. Sadjadi, Phys. Rev. D **73**, 063525 (2006).
- [41] S. Nojiri, S.D. Odintsov, Gen. Relativ. Gravit. **38**, 1285 (2006).
- [42] R.M. Wald, Phys. Rev. D **48**, 3427 (1993).
- [43] A. Sheykhi, AM. Jamil, Gen. Relativ. Gravit. **43**, 2661 (2011).
- [44] N. Radicella, D. Pavón, J. Phys. Conf. Ser. **314**, 012036 (2011).
- [45] M. Jamil, M.U. Farooq, J. Cosmol. Astropart. Phys. **1003**, 001 (2010).
- [46] K. Karami, M. Jamil, N. Sahraei, Phys. Scr. **82**, 045901 (2010).
- [47] R.X. Miao, M. Lib, Y.G. Miaoc, J. Cosmol. Astropart. Phys. **1111**, 033 (2011).
- [48] D. Bak, S.J. Rey, Class. Quantum Grav. **17**, L83 (2000).
- [49] M. Li, Phys. Lett. B **603**,1 (2004).
- [50] S. Hod, Class. Quantum Grav. **21** L97 (2004).
- [51] M. Sharif, S. Rani, Astrophys. Space Sci. **346**, 573 (2013).